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# Improved light-cone sum rules for the electromagnetic form factors of the nucleon

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## Abstract

We calculate the electromagnetic form factors of the nucleon within the light-cone sum rule approach. In comparison to previous work [Phys. Rev. D 65 (2002) 074011] we suggest to use a pure isospin-1/2 interpolating field for the nucleon, since the Chernyak–Zhitnitsky current leads to numerically large, unphysical, isospin violating contributions. The leading-order sum rules are derived for the form factors and the results are confronted with the experimental data. Our approach tends to favor the nucleon distribution amplitudes that are not far from the asymptotic shape.

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**1.** The elastic scattering of electrons off nucleons at momentum transfer  $-Q^2$  is described by the famous Rosenbluth formula [2]

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \left[ \frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1 + \tau} + 2\tau G_M^2(Q^2) \tan^2 \frac{\theta}{2} \right], \quad (1)$$

where  $G_E(Q^2)$  and  $G_M(Q^2)$  are the electric and magnetic Sachs form factors,  $\tau = Q^2/(4m^2)$ ,  $m$  is the nucleon mass and  $\theta$  is the scattering angle in the laboratory frame.  $(d\sigma/d\Omega)_{\text{Mott}}$  is the Mott cross section, which describes the scattering of a pointlike particle. The normalization of the form factors at  $Q^2 = 0$  is given by the nucleon charges and magnetic moments (in units of the nuclear magneton,  $\mu_N = e/2m_p$ ):

$$\begin{aligned} \text{Proton:} \quad G_E(0) &= 1, & G_M(0) &= \mu_p = 2.792847337(29) \quad [3], \\ \text{Neutron:} \quad G_E(0) &= 0, & G_M(0) &= \mu_n = -1.91304272(45) \quad [3]. \end{aligned} \quad (2)$$

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In the Breit frame  $G_E(Q^2)$  and  $G_M(Q^2)$  can be interpreted as the Fourier transforms of the charge distribution and magnetization density in the nucleon, respectively. The matrix element of the electromagnetic current ( $j_\mu^{\text{em}}(x) = e_u \bar{u}(x) \gamma_\mu u(x) + e_d \bar{d}(x) \gamma_\mu d(x)$ ) taken between two nucleon states is conventionally written in terms of the Dirac and Pauli form factors  $F_1(Q^2)$  and  $F_2(Q^2)$ , respectively.

$$\langle P - q | j_\mu^{\text{em}}(0) | P \rangle = \bar{N}(P - q) \left[ \gamma_\mu F_1(Q^2) - i \frac{\sigma_{\mu\nu} q^\nu}{2m} F_2(Q^2) \right] N(P), \quad (3)$$

where  $P_\mu$  is the four-momentum in the initial nucleon state,  $m$  is the nucleon mass,  $P^2 = (P - q)^2 = m^2$ ,  $q_\mu$  is the (outgoing) photon momentum,  $Q^2 = -q^2$ ,  $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$  and  $N(P)$  is the spinor of the nucleon. The electric and magnetic Sachs form factors are related to the Dirac and Pauli form factors in the following way

$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2), \quad G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4m^2} F_2(Q^2). \quad (4)$$

It is known that the experimental data for  $G_M(Q^2)$  at values of  $Q^2$  up to 5 GeV<sup>2</sup> are very well described by the famous dipole formula both for the proton [4–9] and for the neutron [10–12] (following [13] we compare our theoretical predictions only with data sets where both forward and backward angle data were taken in the same apparatus).

$$\frac{1}{\mu_p} G_M^p(Q^2) \sim \frac{1}{\mu_n} G_M^n(Q^2) \sim \frac{1}{(1 + Q^2/\mu_0^2)^2} = G_D(Q^2), \quad \mu_0^2 \sim 0.71 \text{ GeV}^2. \quad (5)$$

For the electric form factor of the proton the experimental situation currently is unclear. Older measurements based on the Rosenbluth separation showed a dipole behavior [5–9] of the electric Sachs form factor, but in recent measurements at the Jefferson Lab Hall A Collaboration using the recoil polarization technique a significant deviation from the dipole was observed [14–16]. This experimental discrepancy has been attracting lots of attention and has not been settled yet (for a review see [13]). The values of the electric form factor of the neutron are very small [10,17,18].

The ultimate goal of the theoretical and experimental analysis of the form factors of the nucleon is the determination of the nucleon wave functions. In recent years it has been becoming increasingly clear that the strict perturbative approach based on QCD factorization and involving at least two hard gluon exchanges is not applicable in the several GeV region and it has to be complemented by some non-perturbative techniques. The method of light-cone sum rules (LCSR) [19] suggests itself since it incorporates both the perturbative and non-perturbative end-point contributions and allows to calculate the form factors as a systematic expansion in terms of nucleon distribution functions of increasing twist [20–22]. Alternative models to determine the form factors of the nucleon can be found, e.g., in [23]. The general concept of the LCSR calculation is familiar from numerous applications of this technique to meson decays [24] and the particular realization for baryons was worked out in Ref. [1]. The starting point of the LCSR approach is that one of the participating nucleons is substituted by a suitable local current. The choice of the current is a subtle issue and is motivated by the necessity to have a strong “nucleon signal” and small sensitivity to the contributions of higher resonances and the continuum. In addition, the choice is influenced by the particular tasks of the calculation. In particular, in [1] the so-called Chernyak–Zhitnitsky (CZ) nucleon current was used since it allows to enhance contributions to the sum rule that are due to the leading-twist distribution amplitude of interest and suppress higher-twist contributions. The essential of this Letter is to point out that the use of the CZ current induces large implicit isospin violations in the sum rules of order 20% (and more) but this deficiency can be overcome by using a modified current which is a pure isospin-1/2 state. In addition to exact isospin symmetry, using the improved current one gets a better stability of the sum rules and a surprisingly good agreement with the experimental data using the set of asymptotic distribution amplitudes. We, therefore, argue that using the pure isospin current is advantageous and allows to increase the accuracy and reliability of the sum rules. Further applications, e.g., to axial form factors will be considered in a subsequent publication [25].

2. We start with the electromagnetic coupling of protons and consider the following correlation function

$$T_v^{\text{em}}(P, q) = i \int d^4x e^{iq \cdot x} \langle 0 | T \{ \eta^P(0) j_v^{\text{em}}(x) \} | P \rangle, \quad (6)$$

which includes an interpolating proton field  $\eta^P$ . The basic principle of sum rules is to calculate this correlation function in two ways and finally compare the two results. First one can insert a complete set of states between  $\eta^P$  and  $j_v^{\text{em}}$  in Eq. (6)

$$T_v^{\text{em}}(P, q) = \sum_{\lambda, s} \langle 0 | \eta^P(0) | \lambda; P - q, s \rangle \frac{1}{m_\lambda^2 - (P - q)^2} \langle \lambda; P - q, s | j_v^{\text{em}}(0) | P \rangle, \quad (7)$$

where  $\lambda$  characterizes the state and  $s$  stands for the polarization. In [1] the CZ current [21]

$$\eta_{\text{CZ}}^P(0) = \varepsilon^{ijk} [u^i(0) C \not{z} u^j(0)] \gamma_5 \not{z} d^k(0) \quad (8)$$

was used for  $\eta^P$ . In this case

$$\langle 0 | \eta^P | P \rangle = f_N(Pz) \not{z} N(P) \quad (9)$$

(here  $z$  is a light-cone vector,  $z^2 = 0$ ), and the coupling  $f_N$  determines the normalization of the leading-twist proton distribution amplitude [20]. Using the definition of the form factors in Eq. (3) the contribution of the nucleon intermediate state in the correlation function Eq. (6) is readily derived to be

$$z^\nu T_\nu(P, q) = \frac{f_N}{m^2 - P'^2} (P'z) \left\{ [2F_1(Q^2)(P'z) - F_2(Q^2)(qz)] \not{z} + F_2(Q^2) \left[ (P'z) + \frac{1}{2}(qz) \right] \frac{\not{q}}{m} \right\} N_P(P), \quad (10)$$

where  $P' = P - q$ . In order to get rid of terms  $\sim z_\nu$  that give subdominant contributions on the light-cone and to simplify the Lorentz structure we contracted the correlation function with  $z^\nu$ . Alternatively, one can calculate the correlation function in Eq. (6) at large Euclidean momenta  $P'^2$  and  $q^2 = -Q^2$  in terms of nucleon distribution amplitudes. To the leading order in the strong coupling one gets expressions of the form (cf. [1])

$$z^\nu T_\nu(P, q) \propto i \int d^4x \int \frac{d^4k}{(2\pi)^4} e^{i(q+k)x} \frac{kz}{k^2} \langle 0 | \epsilon^{ijk} u_i^\alpha(a_1x) u_j^\beta(a_2x) d_k^\gamma(a_3x) | P \rangle C_{\alpha\beta\gamma}, \quad (11)$$

where  $C_{\alpha\beta\gamma}$  are certain coefficients (involving Lorentz structures) and the real numbers  $a_i$  are either one or zero. By assumption  $x^2 \sim 1/(P - q)^2 \rightarrow 0$  and in this limit the remaining three-quark operator sandwiched between the proton state and the vacuum can be written in terms of the three-quark nucleon distribution amplitudes of different twist  $t = 3, 4, 5, 6$ , see [20–22].

$$\langle 0 | \epsilon^{ijk} u_i^\alpha(a_1x) u_j^\beta(a_2x) d_k^\gamma(a_3x) | P \rangle = \sum_i F^{(i)} X^{\alpha\beta} Y^\gamma, \quad (12)$$

where  $F^{(i)} = V^{(i)}, A^{(i)}, T^{(i)}$  are vector, axial-vector and tensor distribution amplitudes and  $X^{\alpha\beta}$  and  $Y^\gamma$  are Dirac structures which are listed in [22]. Equating Eq. (10) and the QCD calculation at a certain intermediate momentum  $(P - q)^2 \sim -1 \text{ GeV}^2$  yields a sum rule for the form factors in terms of the nucleon distribution amplitudes. The matching procedure involves several technical steps that are common for the QCD sum rule approach in general and have the purpose of suppressing contributions both from higher resonances and the continuum, and of higher-twist operators. In particular a Borel transformation is performed, introducing the Borel parameter  $M_B$  instead of  $(P - q)^2$ , and the nucleon contribution is defined by introducing a cutoff in the spectral density at  $s_0 \approx (1.5 \text{ GeV})^2$  which is approximately the mass of the Roper resonance. The Borel parameter  $M_B$  is chosen to be in the range 1.0–1.5 GeV, see [1,24] for details.

The nucleon distribution amplitudes that provide the necessary non-perturbative input to the sum rules are usually written in terms of the conformal expansion [22,26]. The so-called *asymptotic* distribution amplitudes

correspond to taking into account the lowest conformal spin only and comparing the sum rule results with the experimental data one may hope to get an estimate for the corrections. In Ref. [1] it is shown that large contributions of higher conformal spins are not welcome by the data (the fact that higher terms of the conformal expansion tend to overestimate the physical result is already known from the pion form factor [27]), but further work is needed in order to make this conclusion quantitative.

The question that we address in this Letter is whether the accuracy of the sum rules can be improved by the choice of the nucleon interpolation current. In particular, we look at the isospin symmetry. The CZ current (8) does not have a definite isospin so that isospin relations between different nucleon distribution amplitudes are imposed as the relations between the corresponding matrix elements. This current has been chosen for the sum rules in [1] because with this choice the coefficients  $C_{\alpha\beta\gamma}$  in (11) are of order one for the contributions of leading-twist distribution amplitudes and are suppressed, generically, by a power of  $M_B^2$  for higher twists (to leading order in the strong coupling). In [25] we will discuss in addition the Ioffe-current [28] and a current suggested by Chung et al. [28]. Using this two currents within the LCSR approach the effect of higher twist will be enhanced, compared to the use of the CZ-current. Therefore we start in this Letter with the CZ-current. The price to pay is, however, that in the sum (7) there are contributions of both isospin-1/2 and isospin-3/2 states, e.g., the  $\Delta$ -resonance. It is usually believed that the isospin separation is not important since isospin-3/2 resonances are separated from the nucleon by a relatively large mass gap and, therefore, sufficiently strongly suppressed by the Borel transformation. One may also speculate that summing over states with different isospin in fact makes the spectral density more smooth and thus improves the duality approximation for the continuum. Our starting observation is that these arguments can be checked by studying the isospin relations for the sum rule predictions. If one determines only the electromagnetic form factors of the nucleon, as it was done in [1], the necessity to fulfill isospin symmetry is hidden. If, however, one determines in addition to  $F_{1,2}^{pp}$  (proton in the initial and final state) and  $F_{1,2}^{nn}$  (neutron in the initial and final state) the form factors  $F_{1,2}^{np}$ , which arise in the vector part of the weak-current ( $j_v^{\text{weak}}(x) = \bar{u}(x)\gamma_\nu(1 - \gamma_5)d(x)$ ) triggering the  $\beta$ -decay, one can show that the isospin relation

$$F_i^{np} = F_i^{pp} - F_i^{nn} \quad \text{for } i = 1, 2 \quad (13)$$

has to hold. Checking whether Eq. (13) holds numerically for the sum rule predictions, we can test the assumption that the contamination by isospin-3/2 contributions in the sum rules is negligible. The corresponding calculations (see [25]) yield the following result: if one uses asymptotic distribution amplitudes, then the isospin sum rule in Eq. (13) is violated by  $\sim 20\%$ . If higher conformal spin contributions of the distribution amplitudes are taken into account, the isospin violations become even larger. In other words, the use of the CZ current  $\eta_{\text{CZ}}$  for the evaluation of the nucleon form factors leads to an unphysical uncertainty of at least 20% induced by the “pollution” of sum rules by the isospin-3/2 contributions.

The problem can be overcome in a rather simple way by using a modified current which is a isospin-1/2 eigenstate. In particular, we suggest to use

$$\eta_I^p(x) = \frac{2}{3}\epsilon^{ijk}([u^i(x)C\not{z}u^j(x)]\gamma_5\not{z}d^k(x) - [u^i(x)C\not{z}d^j(x)]\gamma_5\not{z}u^k(x)), \quad (14)$$

which is an isospin-1/2 eigenstate and it projects on the leading-twist distribution amplitudes as well so that all “good” properties of the CZ current are retained. The factor 2/3 in Eq. (14) is introduced to fulfill the same normalization condition (9), so that the “hadronic” part of the sum rule (10) remains intact. On the other hand, using the improved current  $\eta_I$  for the quark level calculation the isospin relations in Eq. (13) are recovered exactly. In order to be able to argue that the modified current in (14) is indeed superior for the LCSR calculations, we still need to check what happens with the sum rule predictions. Since in [1] it was found that large corrections to the asymptotic distribution amplitudes seem to be in contradiction to the data, in this Letter we only consider asymptotic distributions as an example. A general case will be studied in [25]. The final LCSRs using the improved

current  $\eta_I$  read

$$\begin{aligned}
 F_1^p = \frac{2e_u}{3f_N} & \left\{ \left( \int_{x_1^0}^1 dx_1 \left[ \rho_1 + \frac{m^2}{M_B^2} (\rho_2 - \rho_3) + \frac{m^4}{M_B^4} \rho_4 \right] (x_1) \text{EXP}_1 \right) \right. \\
 & + \left[ \left( \rho_2 - \rho_3 + \frac{m^2}{M_B^2} \rho_4 \right) (x_1^0) - m^2 \frac{d}{dx_1} \frac{x_1^2 \rho_4(x_1)}{Q^2 + x_1^2 m^2} \Big|_{x_1=x_1^0} \right] \frac{m^2 (x_1^0)^2}{Q^2 + (x_1^0)^2 m^2} \text{EXP}_2 \Big\} \\
 & + \frac{1}{3f_N} e_d \{x_1 \rightarrow x_3, u \rightarrow d\},
 \end{aligned} \tag{15}$$

$$\begin{aligned}
 F_2^p = \frac{4e_u}{3f_N} & \left\{ -\frac{m^2}{M_B^2} \left( \int_{x_1^0}^1 \frac{dx_1}{x_1} \left[ \rho_2 + \frac{m^2}{M_B^2} \rho_4 \right] (x_1) \text{EXP}_1 \right) \right. \\
 & - \left[ \left( \rho_2 + \frac{m^2}{M_B^2} \rho_4 \right) (x_1^0) - x_1^0 m^2 \frac{d}{dx_1} \frac{x_1 \rho_4(x_1)}{Q^2 + x_1^2 m^2} \Big|_{x_1=x_1^0} \right] \frac{m^2 x_1^0}{Q^2 + (x_1^0)^2 m^2} \text{EXP}_2 \Big\} \\
 & + \frac{2}{3f_N} e_d \{x_1 \rightarrow x_3, u \rightarrow d\},
 \end{aligned} \tag{16}$$

where for asymptotic distribution amplitudes

$$\begin{aligned}
 \text{EXP}_1 &:= \exp\left(-\frac{1-x_1}{x_1} \frac{Q^2}{M_B^2} + x_1 \frac{m^2}{M_B^2}\right), & \text{EXP}_2 &= \exp\left(-\frac{s_0 - m^2}{M_B^2}\right), \\
 \rho_1(x) &= 60(1-x)^3 x f_N, \\
 \rho_2(x) &= \frac{1}{18} (1-x)^2 [6x(1-4x)\lambda_1 + (36-370x+1006x^2-117x^3)f_N], \\
 \rho_3(x) &= -\frac{1}{72} (1-x)^3 x [8(9\lambda_1-2\lambda_2)-3(565-417x)f_N], \\
 \rho_4(x) &= \frac{1}{180} (1-x)^3 x^2 [48\lambda_1-5(343-15x)f_N], \\
 x_i^0 &= \frac{1}{2m} \left[ \sqrt{(Q^2 + s_0 - m^2)^2 + 4m^2 Q^2} - (Q^2 + s_0 - m^2) \right].
 \end{aligned} \tag{17}$$

The final result depends on the two ratios  $\lambda_1/f_N$  and  $\lambda_2/f_N$  of the non-perturbative parameters  $f_N = (5.3 \pm 0.5) \times 10^{-3} \text{ GeV}^2$ ,  $\lambda_1 = -(2.7 \pm 0.9) \times 10^{-2} \text{ GeV}^2$  and  $\lambda_2 = (5.1 \pm 1.9) \times 10^{-2} \text{ GeV}^2$ , which are discussed, e.g., in [22].

**3.** The comparison of the sum rule results (15), (16) with the experimental data is shown in Figs. 1–5. In all cases the central value of the LCSR prediction is shown by the solid curve while dashed curves show the effect of the variation of the normalization  $\lambda_1/f_N$  in the range  $-5.1 \pm 1.7$  which is representative of the possible uncertainty. Varying the Borel parameter  $M_B$  in the range of 1.2 GeV to 1.6 GeV yielded no sizeable effect; in the plots  $M_B = \sqrt{2} \text{ GeV}$  is used.

In Fig. 1 we plotted the magnetic form factor of the proton normalized to the dipole formula. In this case the difference compared to using the CZ current appears to be small and our results are close to [1]. In both calculations the LCSR prediction using asymptotic distribution amplitudes tends to overestimate the form factor by about 50%. This disagreement may signal that contributions of higher conformal spin have to be included, but in order to make

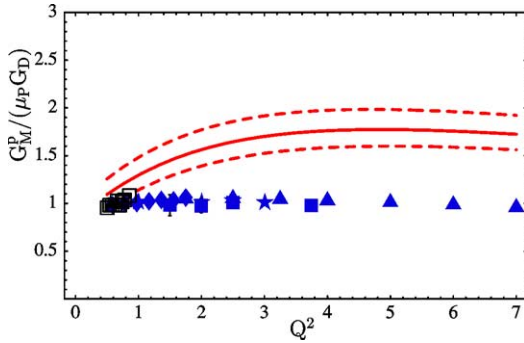


Fig. 1. Solid line: LCSR prediction for the magnetic form factor of the proton normalized to the dipole form factor  $G_M^p / (\mu_p G_D)$ . Dashed lines: errors due to the variation of the normalization  $\lambda_1/f_N$ . Symbols: experimental values: ★: SLAC 1994 [9]; ▲: SLAC 1994 [8]; ■: SLAC 1970 [6]\*; ◆: Bonn 1971 [7]\*; □: Stanford 1966 [5]\* (\*: data actually taken from [13].)

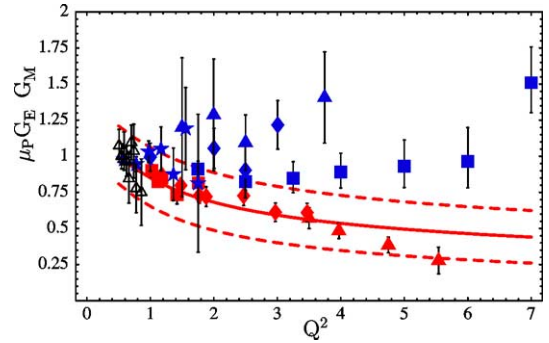


Fig. 2. Solid line: LCSR prediction for the ratio of the electric and magnetic form factors of the proton  $\mu_p G_E^p(Q^2)/G_M^p(Q^2)$ . Dashed lines: errors due to the variation of the normalization  $\lambda_1/f_N$ . Grey (red in the web version) symbols: experimental values obtained via polarization transfer: ▲: Jefferson Lab 2002 [16]; ■: Jefferson Lab 2001 [15]; ◆: Jefferson Lab 2000 [14]; Black (blue in the web version) symbols: experimental values obtained via Rosenbluth separation: ■: SLAC 1994 [8]; ◆: SLAC 1994 [9]; ▲: SLAC 1970 [6]\*; ★: Bonn 1971 [7]\*; △: Stanford 1966 [5]\*. (\*: data actually taken from [13].)

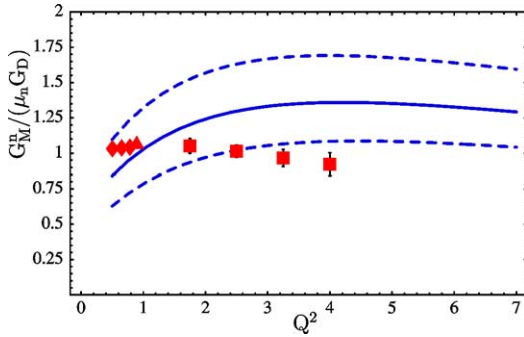


Fig. 3. Solid line: LCSR prediction for the magnetic form factor of the neutron normalized to the dipole form factor  $G_M^n(Q^2)/(\mu_n G_D(Q^2))$ . Dashed lines: errors due to the variation of the normalization  $\lambda_1/f_N$ . Symbols: experimental values: ■: SLAC 1993 [10]; ▲: Mainz 2002 [12]; ◆: Mainz 1998 [11].

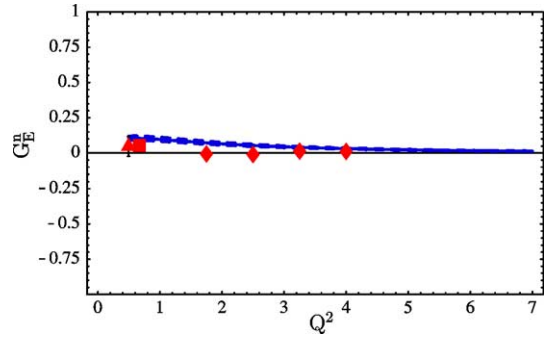


Fig. 4. Solid line: LCSR prediction for the electric form factor of the neutron  $G_E^n(Q^2)$ . Dashed lines: errors due to the variation of the normalization  $\lambda_1/f_N$ . Symbols: experimental values: ◆: SLAC 1993 [10]; ▲: Jefferson Lab 2001 [17]; ■: Mainz 1999 [18].

quantitative statements one first has to calculate perturbative  $\mathcal{O}(\alpha_s)$  corrections to the sum rules which is beyond the tasks of this Letter. The ratio of the electric and the magnetic proton form factors is shown in Fig. 2. Here the LCSR prediction is surprisingly close to the experimental values and tends to favor the values obtained by the recent experiments at Jefferson Lab [14–16]. However, in this case as well, without the inclusion of  $\alpha_s$ -corrections it is premature to draw definite conclusions. The difference to the calculation in [1] is quite sizeable for this ratio, up to 50%. In Figs. 3 and 4 the magnetic and the electric form factors of the neutron are plotted, respectively. The LCSR prediction tends to overestimate the magnetic form factor by about 25% while for the electric form factor both the experiment and the LCSR give comparable small values. In this cases we again observe a noticeable improvement compared to [1]. Finally, in Fig. 5 we study the ratio  $F_2/F_1$  for the proton multiplied by  $Q$ . We actually plotted  $Q F_2/(\kappa_p F_1)$ , with the anomalous magnetic moment of proton  $\kappa_p$ , in order to have the same normalization as the figures in [16]. The LCSR calculation shows a very weak dependence of this ratio on  $Q^2$  which agrees with

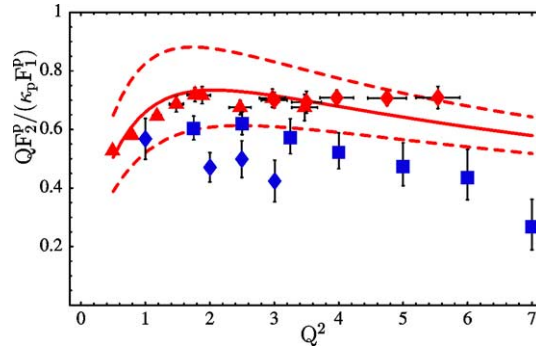


Fig. 5. Solid line: LCSR prediction for the ratio  $QF_2^p(Q^2)/(\kappa_p F_1^p(Q^2))$ . Dashed lines: errors due to the variation of the normalization  $\lambda_1/f_N$ . Grey (red in the web version) symbols: experimental values obtained via polarization transfer:  $\blacktriangle$ ,  $\blacklozenge$ : M. Jones (private communication); Black (blue in the web version) symbols: experimental values obtained via Rosenbluth separation:  $\blacksquare$ : SLAC 1994 [8];  $\blacklozenge$ : SLAC 1994 [9].

the scaling observed at Jefferson Lab [14–16]. In the LCSR approach such behavior results from an interplay of soft and hard contributions with different scale dependence and only holds approximately in a limited range of the momentum transfer. In comparison to the result of the calculation in [1] for  $QF_2/(\kappa_p F_1)$ , which is presented in [29], we are much closer to the experiment now.

To summarize, in this Letter we have presented arguments for the use of the improved nucleon current (14) in the LCSR calculations. Our current retains all desired properties of the CZ current and in addition it fulfills all isospin relations between form factors exactly. Our numerical estimates demonstrate that using the improved current one eliminates an implicit uncertainty of the calculations in [1] that is due to the isospin symmetry violation and also in all cases we obtain a better stability of LCSRs and a better agreement with the data using the set of asymptotic three-quark nucleon distribution amplitudes up to twist-6 constructed in [22]. More details and the application to nucleon axial form factors will be considered in a forthcoming publication [25]. It has to be mentioned that the LCSRs to leading-order accuracy in the QCD coupling only take into account contributions of “soft” or “end-point” regions that are subleading in the true  $Q^2 \rightarrow \infty$  limit. The leading contributions appear at the level of perturbative corrections to the sum rules and their evaluation presents an important task for further studies. We believe that LCSRs with radiative corrections included can provide quantitative information on nucleon distribution amplitudes.

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